

INTEGRACIÓN POR SUSTITUCIÓN

A fin de derivar $\frac{1}{10}(1+x^2)^{10}$ se aplica la regla de la cadena

$$Dx \left[\frac{1}{10}(1+x^2)^{10} \right] = \frac{10}{10}(1+x^2)^9(2x) = 2x(1+x^2)^9$$

Para integrar $2x(1+x^2)^9$ introducimos una nueva variable. Se cambia x por u

Regla de la sustitución: si $u = g(x)$ es una función diferenciable cuyo rango es un intervalo I , y f es continua sobre I , entonces

$$\int f(g(x))g'(x)dx = \int f(u)du$$

Ejemplos: Integrar las siguientes funciones

1. $\int 2x(1+x^2)^9 dx$

Sustitución

$$u = 1 + x^2$$

$$du = 2x dx \quad \Rightarrow \quad \frac{du}{2x} = dx$$

$$\int 2x(1+x^2)^9 dx = \int \cancel{2x}u^9 \frac{du}{\cancel{2x}} = \int u^9 du = \frac{u^{10}}{10} = \boxed{\frac{1}{10}(1+x^2)^{10} + c}$$

2. $\int \frac{x}{\sqrt{1-x^2}} dx$

Sustitución

$$u = 1 - x^2$$

$$du = -2x dx \quad \Rightarrow \quad -\frac{du}{2x} = dx$$

$$\int \frac{x}{\sqrt{1-x^2}} dx = -\frac{1}{2} \int \frac{\cancel{x} du}{u^{1/2} \cancel{x}} = -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \frac{u^{1/2}}{1/2} + c = u^{1/2} + c = \boxed{\sqrt{1-x^2} + c}$$

3. $\int e^{2x^3-5} x^2 dx$

Sustitución

$$u = 2x^3 - 5$$

$$du = 6x^2 dx \quad \Rightarrow \quad \frac{du}{6x^2} = dx$$

$$\int e^{2x^3-5} x^2 dx = \int e^u \cancel{x^2} \frac{du}{\cancel{6x^2}} = \frac{1}{6} \int e^u du = \frac{e^u}{6} + c = \boxed{\frac{1}{6} e^{2x^3-5} + c}$$

Resolver los siguientes ejercicios

Ejercicio	Respuesta	Ejercicio	Respuesta
1. $\int 2x\sqrt{1+x^2} dx$	$\frac{2}{3}\sqrt{(1+x^2)^3} + c$	2. $\int x^3 \cos(x^4 + 2) dx$	$\frac{1}{4} \operatorname{sen}(x^4 + 2) + c$
3. $\int \sqrt{2x+1} dx$	$\frac{1}{3}\sqrt{(2x+1)^3} + c$	4. $\int \frac{x}{\sqrt{1-4x^2}} dx$	$-\frac{1}{4}\sqrt{1-4x^2} + c$
5. $\int \sqrt{3x+4} dx$	$\frac{2}{9}\sqrt{(3x+4)^3} + c$	6. $\int x^2(5+2x^3)^8 dx$	$\frac{1}{54}(5+2x^3)^9 + c$
7. $\int x \cos x^2 dx$	$\frac{1}{2} \operatorname{sen} x^2 + c$	8. $\int \frac{4x^2}{(1-8x^3)^4} dx$	$\frac{1}{18(1-8x^3)^3} + c$
9. $\int x^2 \sqrt{1+x} dx$	*	10. $\int \frac{\operatorname{sen} \sqrt{x}}{\sqrt{x}} dx$	$-2 \cos \sqrt{x} + c$
11. $\int \operatorname{sen} x \sqrt{1-\cos x} dx$	$\frac{2}{3}\sqrt{(1-\cos x)^3} + c$	12. $\int \tan x \sec^2 x dx$	$\frac{1}{2} \tan^2 x + c$
13. $\int \sqrt{1-4y} dy$	$-\frac{1}{6}\sqrt{(1-4y)^3} + c$	14. $\int x^2(x^3-1)^{10} dx$	$\frac{1}{33}(x^3-1)^{11} + c$
15. $\int (x^2-4x+4)^{\frac{4}{3}} dx$	$\frac{3}{11}\sqrt[3]{(x-2)^{11}} + c$	16. $\int x\sqrt{x+1} dx$	$\frac{2}{5}\sqrt{(x+1)^5} - \frac{2}{3}\sqrt{(x+1)^3}$
17. $\int \sqrt{3-2x} x^2 dx$	**	18. $\int \cos 4\theta d\theta$	$\frac{1}{4} \operatorname{sen} 4\theta + c$
19. $\int e^{kx} dx$	$\frac{1}{k} e^{kx} + c$	20. $\int \frac{1}{2} t \operatorname{sen} 4t^2 dt$	$-\frac{1}{16} \cos 4t^2 + c$
21. $\int \cos x(2+\operatorname{sen} x)^5 dx$	$\frac{1}{6}(2+\operatorname{sen} x)^6 + c$	22. $\int \sqrt{1+\frac{1}{3x}} \frac{dx}{x^2}$	$-2\sqrt{\left(1+\frac{1}{3x}\right)^3} + c$
23. $\int 2\operatorname{sen} x \sqrt[3]{1+\cos x} dx$	$-\frac{3}{2}\sqrt[3]{(1+\cos x)^4} + c$	24. $\int \operatorname{sen}^3 \theta \cos \theta d\theta$	$\frac{1}{4} \operatorname{sen}^4 \theta + c$
25. $\int \frac{(y+3)dy}{(3-y)^{\frac{2}{3}}}$	$-18\sqrt[3]{(3-y)} + \frac{3}{4}\sqrt[3]{(3-y)^4}$	26. $\int \frac{\sec^2 3\sqrt{t}}{\sqrt{t}} dt$	$\frac{3}{2} \tan 3\sqrt{t} + c$
27. $\int \frac{x^3}{\sqrt{1-2x^2}} dx$	***	28. $\int \tan x dx$	$-\ln \cos x + c$ o $\ln \sec x + c$
29. $\int \sec x dx$	$\ln \sec x + \tan x + c$	30. $\int \cot x dx$	$\ln \operatorname{sen} x + c$
31. $\int \csc x dx$	$\ln \csc x - \cot x + c$	32. $\int \frac{y^3}{(1-2y^4)^5} dy$	$\frac{1}{32(1-2y^4)^4} + c$
33. $\int \frac{2r}{(1-r)^{\frac{2}{3}}} dr$	$\frac{3}{2}\sqrt[3]{(1-r)^4} - 6\sqrt[3]{1-r} + c$	34. $\int \frac{x^2+2x}{\sqrt{x^3+3x^2+1}} dx$	$\frac{2}{3}\sqrt{x^3+3x^2+1} + c$
35. $\int \frac{2x+1}{x^2+x+1} dy$	$\ln x^2+x+1 + c$	36. $\int \left(t+\frac{1}{t}\right)^{\frac{3}{2}} \left(\frac{t^2-1}{t^2}\right) dt$	$\frac{2}{5}\sqrt{\left(t+\frac{1}{t}\right)^5} + c$

37. $\int \frac{s}{2s+3} ds$	$\frac{1}{4}(2s+3) - \frac{3}{4} \ln 2s+3 + c$	38. $\int x(x^2+9)^{1/2} dx$	$\frac{1}{3} \sqrt{(x^2+9)^3} + c$
39. $\int \frac{(x^2+2)}{(x^3+6x-1)} dx$	$\frac{1}{3} \ln x^3+6x-1 + c$	40. $\int \frac{3x^2-6}{\sqrt{x^3-6x}} dx$	$2\sqrt{x^3-6x} + c$
41. $\int x^2 \operatorname{sen} x^3 dx$	$-\frac{1}{3} \cos x^3 + c$	42. $\int \operatorname{sen} 3y dy$	$-\frac{1}{3} \cos 3y + c$
43. $\int (2x+7)^5 dx$	$\frac{1}{12}(2x+7)^6 + c$	44. $\int \frac{1}{e^{-y}+1} dy$	$\ln 1+e^y + c$
45. $\int \frac{s}{s+1} ds$	$(s+1) - \ln s+1 + c$	46. $\int \frac{x^2}{(x-1)^3} dx$	$\ln x-1 - \frac{2}{x-1} - \frac{1}{2(x-1)^2}$
47. $\int \cos^3 4x \operatorname{sen} 4x dx$	$-\frac{1}{16} \cos^4 4x + c$	48. $\int x^2 \sqrt{x^3+5} dx$	$\frac{2}{9} \sqrt{(x^3+5)^3} + c$
49. $\int \frac{1}{\sqrt{5x-3}} dx$	$\frac{2}{5} \sqrt{5x-3} + c$	50. $\int \frac{2x}{\sqrt[3]{6-5x^2}} dx$	$-\frac{3}{10} \sqrt[3]{(6-5x^2)^2} + c$
51. $\int \frac{\operatorname{sen} x}{\sqrt{\cos^3 x}} dx$	$\frac{2}{\sqrt{\cos x}} + c$	52. $\int \frac{e^x}{\sqrt{e^x-5}} dx$	$2\sqrt{e^x-5} + c$
53. $\int \frac{\operatorname{sen} 2x}{\sqrt{\cos 2x}} dx$	$-\sqrt{\cos 2x} + c$	54. $\int \frac{(e^x + \operatorname{sen} x)}{\sqrt{e^x - \cos x}} dx$	$2\sqrt{e^x - \cos x} + c$
55. $\int \frac{(x+2)}{\sqrt{x^2+4x+1}} dx$	$\sqrt{x^2+4x+1} + c$	56. $\int \sqrt[3]{2-3x} dx$	$-\frac{1}{4} \sqrt[3]{(2-3x)^4} + c$
57. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$	$-\frac{1}{2} e^{\sqrt{x}} + c$	58. $\int 6e^{3x} dx$	$2e^{3x} + c$
59. $\int \frac{1}{x \ln x } dx$	$\ln \ln x + c$	60. $\int (12x^2+4)(4x^3+4x)^3 dx$	$\frac{1}{4} (4x^3+4x)^4 + c$

$$* \frac{2}{7} \sqrt{(1+x)^7} - \frac{4}{5} \sqrt{(1+x)^5} + \frac{2}{3} \sqrt{(1+x)^3} + c$$

$$** -\frac{3}{4} \sqrt{(3-2x)^3} + \frac{3}{10} \sqrt{(3-2x)^5} - \frac{1}{28} \sqrt{(3-2x)^7} + c$$

$$*** -\frac{1}{4} \sqrt{1-2x^2} + \frac{1}{12} \sqrt{(1-2x^2)^3} + c$$