

INTEGRACIÓN DE FUNCIONES ALGEBRAICAS MEDIANTE SUSTITUCIÓN TRIGONOMÉTRICA

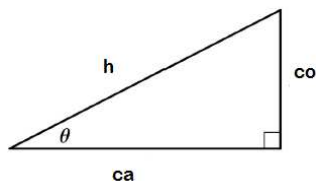
A menudo es posible hallar la antiderivada de una función cuando el integrando presenta expresiones de la forma:

Expresión en el integrando	Sustitución trigonométrica
$\sqrt{a^2 - x^2}$	$x = a \operatorname{sen} \theta \quad x > 0$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta \quad x > 0$
$\sqrt{x^2 - a^2}$	$x = a \operatorname{sec} \theta \quad x > 0$

Ahora observe las simplificaciones que realizan estas sustituciones

$$\begin{aligned}\sqrt{a^2 - x^2} &= \sqrt{a^2 - a^2 \operatorname{sen}^2 \theta} = \sqrt{a^2 \cos^2 \theta} = a \cos \theta \\ \sqrt{a^2 + x^2} &= \sqrt{a^2 + a^2 \tan^2 \theta} = \sqrt{a^2 \sec^2 \theta} = a \sec \theta \\ \sqrt{x^2 - a^2} &= \sqrt{a^2 \sec^2 \theta - a^2} = \sqrt{a^2 \tan^2 \theta} = a \tan \theta\end{aligned}$$

Identidades útiles en esta sección



$$\operatorname{sen} \theta = \frac{co}{h}$$

$$\cos \theta = \frac{ca}{h}$$

$$\tan \theta = \frac{co}{ca}$$

$$\sec \theta = \frac{h}{ca}$$

$$\operatorname{csc} \theta = \frac{h}{co}$$

$$\cot \theta = \frac{ca}{co}$$

Ángulo doble

$$\operatorname{sen} 2\theta = 2 \operatorname{sen} \theta \cos \theta$$

Resolver los siguientes ejercicios

Ejercicio	Respuesta	Ejercicio	Respuesta
1. $\int \frac{1}{\sqrt{1-x^2}} dx$	$\operatorname{sen}^{-1} x + c$	2. $\int \frac{\sqrt{9-x^2}}{x^2} dx$	$-\frac{\sqrt{9-x^2}}{x} - \operatorname{sen}^{-1} \frac{x}{3} + c$
3. $\int \frac{1}{x^3 \sqrt{x^2-9}} dx$	$\frac{1}{54} \operatorname{sen}^{-1} \frac{\sqrt{x^2-9}}{x} + \frac{\sqrt{x^2-9}}{18x^2} + c$	4. $\int \sqrt{x^2+5} dx$	$\frac{x\sqrt{x^2+5}}{2} + \frac{5}{2} \ln \left \frac{x+\sqrt{x^2+5}}{\sqrt{5}} \right $

5. $\int \frac{1}{x^2 \sqrt{4-x^2}} dx$	$-\frac{\sqrt{4-x^2}}{4x} + c$	6. $\int \frac{x^2}{\sqrt{1-x^2}} dx$	$\frac{1}{2} \operatorname{sen}^{-1} x - \frac{x\sqrt{1-x^2}}{2} + c$
7. $\int \frac{\sqrt{25-x^2}}{x} dx$	$5 \ln \left \frac{5-\sqrt{25-x^2}}{x} \right + \sqrt{25-x^2} + c$	8. $\int \sqrt{x^2+4} dx$	$\frac{x\sqrt{x^2+4}}{2} + 2 \ln \left \frac{x+\sqrt{x^2+4}}{2} \right $
9. $\int \sqrt{x^2+7} dx$	$\frac{x\sqrt{x^2+7}}{2} + \frac{7}{2} \ln \left \frac{x+\sqrt{x^2+7}}{\sqrt{7}} \right $	10. $\int \frac{x^2}{\sqrt{x^2+6}} dx$	$\frac{x\sqrt{x^2+6}}{2} - 3 \ln \left \frac{x+\sqrt{x^2+6}}{\sqrt{6}} \right $
11. $\int \frac{1}{x^2 \sqrt{9-x^2}} dx$	$-\frac{\sqrt{9-x^2}}{9x} + c$	12. $\int \frac{\sqrt{4-x^2}}{x^2} dx$	$-\frac{\sqrt{4-x^2}}{x} - \operatorname{sen}^{-1} \frac{x}{2} + c$
13. $\int \frac{1}{x\sqrt{x^2+4}} dx$	$\frac{1}{2} \ln \left \frac{\sqrt{x^2+4}-2}{x} \right + c$	14. $\int \frac{x}{\sqrt{x^2-25}} dx$	$\sqrt{x^2-25} + c$
15. $\int \frac{\sqrt{4-x^2}}{x} dx$	$2 \ln \left \frac{2-\sqrt{4-x^2}}{x} \right + \sqrt{4-x^2} + c$	16. $\int \frac{1}{\sqrt{9+x^2}} dx$	$\ln \left \frac{\sqrt{9+x^2}+x}{3} \right + c$
17. $\int \frac{1}{x^2 \sqrt{x^2+4}} dx$	$-\frac{\sqrt{x^2+4}}{4x} + c$	18. $\int \frac{x}{\sqrt{x^2+4}} dx$	$\sqrt{x^2+4} + c$
19. $\int \frac{1}{x^2 \sqrt{x^2-9}} dx$	$\frac{\sqrt{x^2-9}}{9x} + c$	20. $\int x^3 \sqrt{9-x^2} dx$	$-3\sqrt{(9-x^2)^3} + \frac{\sqrt{(9-x^2)^5}}{5} + c$
21. $\int \frac{x^3}{\sqrt{x^2+9}} dx$	$\frac{\sqrt{(x^2+9)^3}}{3} - 9\sqrt{x^2+9} + c$	22. $\int \frac{x^3}{\sqrt{16-x^2}} dx$	$-16\sqrt{16-x^2} + \frac{\sqrt{(16-x^2)^3}}{3} + c$
23. $\int \frac{1}{t^3 \sqrt{t^2-1}} dt$	$\operatorname{sen}^{-1} \frac{\sqrt{t^2-1}}{t} + \frac{\sqrt{t^2-1}}{2t^2} + c$	24. $\int x^3 \sqrt{x^2+4} dx$	$\frac{\sqrt{(x^2+4)^5}}{5} - \frac{4\sqrt{(x^2+4)^3}}{3} + c$
25. $\int \frac{1}{x^2 \sqrt{25-x^2}} dx$	$-\frac{\sqrt{25-x^2}}{25x} + c$	26. $\int \frac{\sqrt{1+x^2}}{x} dx$	$\ln \left \frac{\sqrt{1+x^2}-1}{x} \right + \sqrt{1+x^2} + c$
27. $\int \frac{t}{\sqrt{25-t^2}} dt$	$-\sqrt{25-t^2} + c$	28. $\int \frac{\sqrt{x^2-4}}{x} dx$	$\sqrt{x^2-4} - 2 \operatorname{sen}^{-1} \frac{\sqrt{x^2-4}}{x} + c$
29. $\int \frac{x^2}{\sqrt{16-x^2}} dx$	$8 \operatorname{sen}^{-1} \frac{x}{4} - \frac{x\sqrt{16-x^2}}{2} + c$	30. $\int \frac{x^2}{\sqrt{x^2-16}} dx$	$\frac{x\sqrt{x^2-16}}{2} + 8 \ln \left \frac{x+\sqrt{x^2-16}}{4} \right $
31. $\int \frac{10}{x^2 \sqrt{25-x^2}} dx$	$-\frac{2\sqrt{25-x^2}}{5x} + c$	32. $\int \frac{x^2}{\sqrt{25-x^2}} dx$	$\frac{25}{2} \operatorname{sen}^{-1} \frac{x}{5} - \frac{x\sqrt{25-x^2}}{2} + c$
33. $\int \frac{x^3}{\sqrt{x^2-4}} dx$	$4\sqrt{x^2-4} + \frac{\sqrt{(x^2-4)^3}}{3} + c$	34. $\int \frac{9x^3}{\sqrt{1+x^2}} dx$	$3\sqrt{(1+x^2)^3} - 9\sqrt{1+x^2} + c$

35. $\int \frac{\sqrt{x^2 - 36}}{x^4} dx$	$\frac{\sqrt{(x^2 - 36)^3}}{108x^3} + c$	36. $\int \frac{1}{\sqrt{4x^2 + 1}} dx$	$\frac{1}{2} \ln \sqrt{4x^2 + 1} + 2x + c$
37. $\int \sqrt{1 - 4x^2} dx$	$\frac{1}{4} \operatorname{sen}^{-1} 2x + \frac{x\sqrt{1 - 4x^2}}{2} + c$	38. $\int \frac{1}{x^2 \sqrt{16x^2 - 9}} dx$	$\frac{\sqrt{16x^2 - 9}}{9x} + c$
39. $\int \frac{x^3}{(4x^2 + 9)^{3/2}} dx$	$\frac{\sqrt{4x^2 + 9}}{16} + \frac{3}{\sqrt{4x^2 + 9}} + c$	40. $\int \frac{1}{(x^2 + 1)^{3/2}} dx$	$\frac{x}{\sqrt{x^2 + 1}} + c$
41. $\int \frac{1}{(6 - x^2)^{3/2}} dx$	$\frac{x}{6\sqrt{6 - x^2}} + c$	42. $\int \frac{1}{(2 + x^2)^{3/2}} dx$	$\frac{x}{2\sqrt{2 + x^2}} + c$
43. $\int \frac{1}{(4x^2 - 9)^{3/2}} dx$	$-\frac{x}{9\sqrt{4x^2 - 9}} + c$	44. $\int \frac{1}{(16 + x^2)^{3/2}} dx$	$\frac{x}{16\sqrt{16 + x^2}} + c$
45. $\int \frac{1}{(w^2 - 4)^{3/2}} dw$	$-\frac{w}{4\sqrt{w^2 - 4}} + c$	46. $\int \frac{t}{(1 - t^2)^{3/2}} dt$	$\frac{1}{\sqrt{1 - t^2}} + c$
47. $\int \frac{1}{(x^2 + 3)^{3/2}} dx$	$\frac{x}{3\sqrt{3 + x^2}} + c$	48. $\int \frac{1}{(1 - t^2)^{3/2}} dt$	$\frac{1}{3} \left(\frac{t}{\sqrt{1 + t^2}} \right)^3 + \frac{t}{\sqrt{1 + t^2}} + c$
49. $\int \frac{1}{\sqrt{x^2 + 2x + 26}} dx$	$\ln \left \frac{\sqrt{x^2 + 2x + 26} + 5}{x + 1} \right + c$	50. $\int \frac{1}{\sqrt{16 + 6x - x^2}} dx$	$\operatorname{sen}^{-1} \frac{x - 3}{5} + c$
51. $\int \sqrt{5 + 4x - x^2} dx$	$\frac{9}{2} \operatorname{sen}^{-1} \frac{x - 2}{3} + \frac{(x - 2)\sqrt{5 + 4x - x^2}}{2}$	52. $\int \frac{1}{\sqrt{x^2 - 6x + 13}} dx$	$\ln \left \frac{\sqrt{x^2 - 6x + 13} + x - 3}{2} \right + c$
53. $\int \frac{1}{\sqrt{9x^2 + 6x - 8}} dx$	$\frac{1}{3} \ln \left \frac{(3x + 1) + \sqrt{9x^2 + 6x - 8}}{3} \right + c$	54. $\int \frac{1}{(x^2 + 8x + 25)^{3/2}} dx$	$\frac{x + 4}{9\sqrt{x^2 + 8x + 25}}$
55. $\int \frac{1}{(5 - 4x - x^2)^{3/2}} dx$	$\frac{(x + 2)^3}{243\sqrt{(5 - 4x - x^2)^3}} + \frac{(x + 2)}{81\sqrt{5 - 4x - x^2}}$	56. $\int \frac{1}{\sqrt{4x + x^2}} dx$	$\ln \left \frac{x + 2 + \sqrt{4x + x^2}}{2} \right + c$
57. $\int \frac{x}{\sqrt{x^2 - 6x + 5}} dx$	$\sqrt{x^2 - 6x + 5} + 3 \ln \left \frac{(x - 3) + \sqrt{x^2 - 6x + 5}}{2} \right + c$		
58. $\int \frac{x^2}{\sqrt{4x - x^2}} dx$	$6 \operatorname{sen}^{-1} \frac{x - 2}{2} - \frac{(x - 2)\sqrt{4x - x^2}}{2} - 4\sqrt{4x - x^2} + c$		
59. $\int \frac{3x}{\sqrt{x^2 + 2x + 5}} dx$	$3\sqrt{x^2 + 2x + 5} - 3 \ln \left \frac{\sqrt{x^2 + 2x + 5} + x + 1}{2} \right + c$		
60. $\int \frac{2x - 1}{\sqrt{x^2 + 4x + 5}} dx$	$2\sqrt{x^2 + 4x + 5} - 5 \ln \left \sqrt{x^2 + 4x + 5} + x + 2 \right + c$		